

UNSTEADY HEAT TRANSFER IN TRANSVERSE AIRFLOW  
PAST A CYLINDER

B. M. Smol'skii, L. A. Sergeeva,  
and M. F. Kupchinova

UDC 536.242

The airflow cooling of a thin-walled hollow copper cylinder is investigated in the range of Reynolds numbers  $Re = 10^4 - 10^5$ .

The results of previous theoretical and experimental studies [1] indicate that the heat-transfer characteristics obtained under unsteady and steady conditions differ appreciably from one another. It has also been shown [1] that the values of the unsteady heat-transfer rates themselves often differ according to the data of different authors. In particular, differences are noted in the results of the investigation of unsteady heat transfer associated with transverse airflow past a cylinder.

Kudryashev and Smirnov [2] have heated a hollow Duralumin cylinder with a diameter of 36 mm and wall thickness of 0.25 to 180°C and inserted it into an airflow. The wall temperature of the cylinder was measured with a copper-Constantan thermocouple; the temperature was assumed, within certain tolerances, to be constant throughout the thickness of the cylinder wall. The heat-transfer coefficient in the process of cooling of the cylinder was determined by the regular regime method. The results were processed in dimensionless-group form. In the interval  $0 < FoRe^{0.7} < 23$

$$\frac{Nu^2}{Nu_{st}^2} = 1 + \frac{3.6}{(FoRe^{0.7})^{0.55}}, \quad (1)$$

and for  $23 < FoRe^{0.7} < 70$

$$\frac{Nu^2}{Nu_{st}^2} = 1 + \frac{288}{(FoRe^{0.7})^2}. \quad (2)$$

It follows from the experimental data that the heat-transfer coefficient under unsteady conditions at the beginning of the cooling process is equal to or greater than twice its steady value (i.e., the value of  $\alpha$  for the same constant freestream parameters and a constant surface temperature equal to the instantaneous value in the unsteady process). Then the ratio  $Nu/Nu_{st}$  decreases, becoming equal to unity at  $FoRe^{0.7} = 70$  for all values of  $Re$  from 1000 to 5000.

It has been noted [3] that the investigation in [2] suffers from shortcomings in the slow response of the measurement circuit and the lack of precautions to prevent heat leakage along the cylinder.

Parnas [3] has studied the process of heating of cylinders with diameters of 8.4 and 2.6 mm made of a material characterized by a low thermal conductivity. The surface temperature of the cylinder was measured with a resistance thermometer whose temperature-sensing element comprised a wire wound on the surface of the cylinder, thereby averaging the recorded temperature over the circumference and length of the cylinder. The cylinder was mounted perpendicular to the axis of an air jet flowing from a nozzle and heated to 80°C. A baffle was initially interposed between the nozzle and the experimental body and was subsequently lifted out after a relatively short time interval. The number  $Re$  was varied from 400 to 3700.

It was shown as a result of the experimental study [3] that the ratio  $Nu/Nu_{st}$  under the

---

A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from *Inzhenerno-Fizicheskii Zhurnal*, Vol. 38, No. 5, pp. 813-819, May, 1980. Original article submitted June 14, 1979.

given conditions increases from zero to a steady-state value. A single-valued influence of the number  $Re$  on the nature of the variation of the heat flow and arrival time at the steady state was not observed. It is also noted that the above-indicated behavior of the ratio  $Nu/Nu_{st}$  is attributable not only to rearrangement of the profiles of the parameters in the boundary layer of the cylinder, but also to stabilization of the flow parameters after removal of the baffle in the zone from the baffle site to the cylinder. The arrival time at the steady state was  $Ho \approx 20-40$  ( $Ho = v\tau/d$ ).

The results of [2, 3] differ considerably in terms of the behavior of the number  $Nu$  with time, the arrival time at the steady state, and the dependence of  $Nu$  on  $Re$ . The nature of the function  $Nu = f(\tau)$  is clearly determined by whether the experimental body is heated or cooled in the flow.

In the present article we give the results of an investigation of the local unsteady heat transfer of a hollow copper cylinder during cooling in an airflow. The heat-transfer coefficients were determined around the perimeter of the cylinder in transverse airflow at points corresponding to values of the angle between the direction opposite to the freestream velocity vector and the cylinder radius  $\varphi = 0.15, 30, 45, 60, 75, 90, 135, 150, 165, 180,$  and  $270^\circ$  in the interval of values  $Re = 10^4-10^5$  for temperature differences between the flow and surface of the cylinder  $\Delta t \approx 0-80^\circ C$ .

Surkov and Vanitskaya [4] have solved the problem of reconstructing an arbitrary heat flow on a surface according to the results of measurements of the temperature at two points of the wall of a thick hollow cylinder. The problem was solved with regard for the temperature dependence of the wall heat capacity and thermal conductivity.

Using the solution obtained in [4], we can determine the heat-transfer coefficient with an arbitrary time variation:

$$\alpha(\tau) = \frac{\lambda F'(R_2, \tau)}{\Phi_2 - t_f}, \quad (3)$$

where  $\Phi_2$  is the experimentally known time dependence of the temperature at  $r = R_2$  and  $F'(R_2, \tau)$  is the derivative of the temperature, determined on the basis of the solution in [4]:

$$F(r, \tau) = \frac{\ln(R_2/r)}{\ln(R_2/R_1)} \psi_1(\tau) + \frac{\ln(r/R_1)}{\ln(R_2/R_1)} \psi_2(\tau) + \psi_1'(\tau) \frac{1}{4a_0e} F_1(r) \sum_{k=1}^{\infty} \frac{1}{\alpha_k (k-1)!} + \psi_2'(\tau) \frac{1}{4a_0e} F_2(r) \sum_{k=1}^{\infty} \frac{1}{\alpha_k (k-1)!}. \quad (4)$$

For a hollow cylinder of a thermally conducting material with small temperature differences across the wall thickness in comparison with the time variation of the wall temperature, the boundary conditions can be determined on the basis of the condition of a small Biot number.

If  $Bi \leq 0.02$ , the quantity of heat admitted to the body in a time interval  $d\tau$  is equal to the variation of the heat capacity of the body:

$$Qd\tau = G c dt = \alpha(\tau) (t - t_f) F d\tau, \quad (5)$$

whence

$$\alpha(\tau) = \frac{R_2^2 - R_1^2}{2R_2} c_p \frac{1}{t - t_f} \frac{dt}{d\tau} = \frac{R_2^2 - R_1^2}{2R_2} c_p \frac{d \ln(t - t_f)}{d\tau}, \quad (6)$$

where  $R_2$  and  $R_1$  are the outside and inside radii of the cylinder and  $t_f$  is the freestream temperature of the fluid.

It is observed experimentally that the parts of the curves  $\vartheta = t - t_f = f(\tau)$ , beginning after a time of 20-40 sec of the process and depending on the position around the perimeter of the cylinder cross section, are close to linear, i.e.,  $\vartheta = \vartheta_1 - k\tau$ . The test duration was 130 sec. In this case the heat-transfer coefficient is equal to

$$\alpha = \frac{R_2^2 - R_1^2}{2R_2} c_p \frac{k}{\vartheta_1 - k\tau}. \quad (7)$$

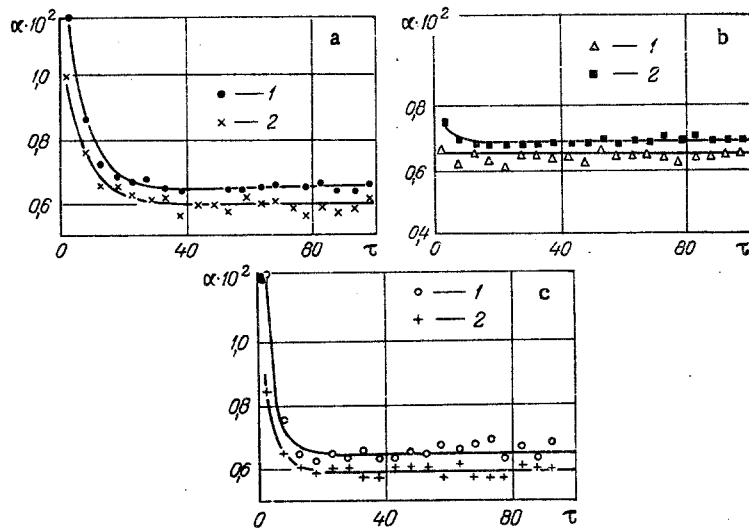


Fig. 1. Heat-transfer coefficient  $\alpha$ ,  $W/cm^2(^{\circ}C)$ , versus time  $\tau$ , sec, for a freestream velocity  $v_{\infty} = 12$  m/sec. a: 1)  $\varphi = 0$ ; 2)  $45^{\circ}$ ; b: 1)  $\varphi = 90^{\circ}$ ; 2)  $270^{\circ}$ ; c: 1)  $\varphi = 180^{\circ}$ ; 2)  $135^{\circ}$ .

the heat-transfer coefficient in cooling of the cylinder in a transverse airflow the time, and for the indicated practical situations this dependence is characterized by expression (7). This expression also describes the dependence of the unsteady heat-transfer coefficient on the thermophysical properties of the material and dimensions of the cylinder.

As  $R_1 \rightarrow R_2$  (thin-walled cylinder) we obtain from (6) an expression for the heat-transfer coefficient for a plate:

$$\alpha(\tau) = c\rho\delta \frac{d \ln(t - t_f)}{d\tau}, \quad (8)$$

where  $\delta$  is the thickness of the plate or wall of the hollow cylinder.

The method described in [5] can also be used to process the experimental data in order to determine the unsteady boundary conditions.

The outside diameter of the experimental cylinder was made equal to 80 mm with regard for the data of a theoretical study [6], according to which the relaxation time of the thermal boundary layer increases with the value of the governing length in an external flow. For the given cylinder dimensions, the degree of flow stagnation in the wind tunnel was such that the Reynolds number correction calculated by the method of Zdanavichus [7] did not exceed a few percent. The inside diameter of the cylinder was 46 mm. The use of copper as the cylinder material provided a ratio between the thermal diffusivities of the body and medium equal to 6, which is sufficient to promote the inception of unsteady effects. The ratio of the length of the working section of the cylinder to the total length (410 mm) was  $\sim 0.1$ . The end heat leakages in the working section were negligible, as confirmed by measurements of the temperature along the length of the cylinder. Estimates showed that the quantity of heat transferred by natural convection is not greater than 3% of the heat flow due to forced convection.

The working section of the cylinder comprised three rings, each with a thickness of 15 mm. To prevent heat leakage around the perimeter and along the length of the cylinder and to ensure one-dimensional heat propagation in accordance with the method of determination of the heat-transfer coefficient, the copper rings were made up of sectors mutually thermally insulated by Teflon spacers. The gaps were smeared on the outside with a cement containing a plastic filler. The circumference of the ring consisted of seven sectors, permitting measurements around the perimeter of the cylinder.

The wall temperature of the cylinder was measured with a Chromel-Copel thermocouple with electrodes 0.2 mm in diameter, which were mounted at a distance of 0.9 mm from the outer surface and on the inner surface of the cylinder. The thermocouple lead was placed within the limits of a sector along isothermal surfaces.

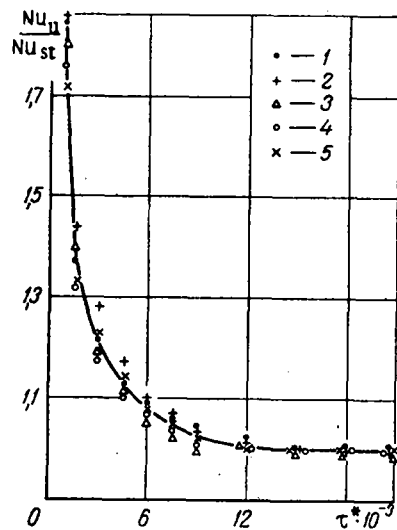


Fig. 2. Ratio of unsteady to steady Nusselt numbers versus parameter  $\tau^*$ ,  $\varphi = 0^\circ$ . 1)  $v_\infty = 5.3$ ; 2) 8.3; 3) 12.0; 4) 14.5; 5) 19.2 m/sec.

Prior to the experiment, the cylinder was heated in a tubular heater with a Nichrome heating element and was kept in the heater. According to measurements at the ends of the cylinder and in the working section, the nonuniformity of the temperature field in the experimental body did not exceed  $0.01^\circ\text{C}$ . The response time of the instrument used to measure the temperature did not exceed 0.01 sec. After the cylinder had been heated to  $80^\circ\text{C}$ , the detainer was released, and the cylinder entered the airflow under its own weight. The entry time was negligible in comparison with the time of the investigated process.

The experimentally obtained temperature curves approach straight lines with time. A deviation from linear is observed in the initial period.

With the use of expression (7), the heat-transfer coefficient was determined according to a routine on an Elektronika S50 computer for time intervals  $\Delta t = 2.5, 5.0$ , and 10 sec. The results of the calculations for different time steps exhibit good agreement. A certain increase in the scatter of the points with diminution of the step is attributable to the increased role of the temperature reading errors in this case. The use of the values of the temperature from expressions piecewise approximating the temperature curves in the calculations significantly reduces the scatter of the values of  $\alpha$ .

The measurements of the heat-transfer coefficient around the perimeter of the cylinder cross section during cooling in a constant-velocity (12 m/sec) constant-temperature ( $20^\circ\text{C}$ ) flow show that (Fig. 1a) in the frontal ("bow") section of the cylinder ( $\varphi = 0-60^\circ$ ) the heat-transfer coefficient decreases with time to a certain constant value [ $\sim 0.62 \cdot 10^{-2} \text{ W/cm}^2(\text{C})$ ]. The variation of the heat-transfer coefficient attains 50% or more at  $\varphi = 0^\circ$ . The period of variation of  $\alpha$  is 30-40 sec. Approximately the same kind of variation of the coefficient takes place in the "aft" part of the cylinder at angles  $\varphi = 135-180^\circ$  (Fig. 1c). At angles of  $90^\circ$  and  $270^\circ$  the heat-transfer coefficient is practically constant from the start of the cooling process (Fig. 1b).

Thus, the variation of  $\alpha$  is a maximum with respect to magnitude and time for fore (in the downstream direction) and aft points of a cylinder in transverse airflow and is close to zero at extreme lateral points.

Measurements for other values of the freestream velocity corroborate the given law in the time variation of the heat-transfer coefficient for various angles between the cylinder radius and the direction opposite to the flow.

Measurements of the heat-transfer coefficient around the perimeter of the cylinder cross section under unsteady conditions for constant values of the velocity from 5 to 20 m/sec have shown that, for example, at an angle  $\varphi = 0^\circ$  the deviation of the heat-transfer coefficient from the quasisteady value attains  $0.2 \cdot 10^{-2} \text{ W/cm}^2(\text{C})$  in the first seconds of the process, independently of the flow velocity. The stabilization time of the heat-transfer coefficient is equal to 30-40 sec and also does not depend on the flow velocity. At an angle of  $180^\circ$  the indicated quantities are equal to  $0.1 \cdot 10^{-2} \text{ W/cm}^2(\text{C})$  and 20-25 sec, respectively, again independently of the flow velocity. At an angle of  $90^\circ$  there are no variations in the heat-transfer coefficient for any of the investigated velocity values. Sample curves of the

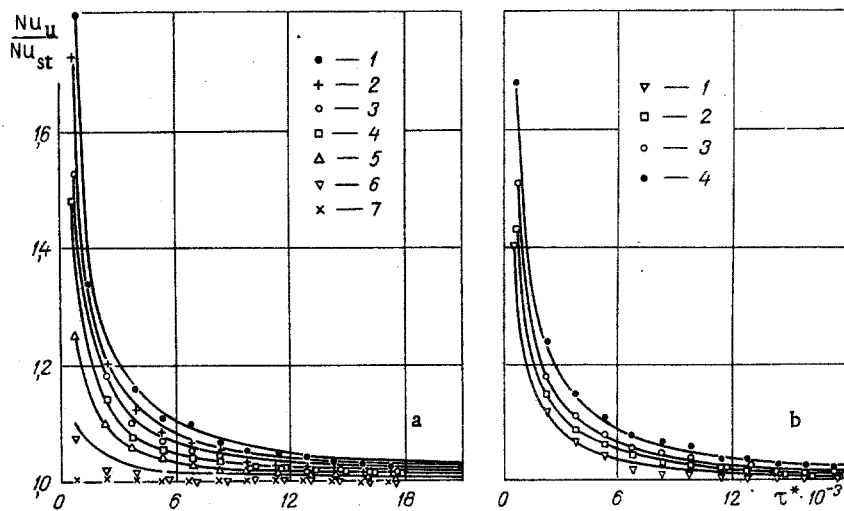


Fig. 3. Ratio of unsteady to steady Nusselt numbers versus parameter  $\tau^*$  for a freestream velocity  $v_\infty = 12$  m/sec. a: 1)  $\varphi = 0^\circ$ ; 2)  $15^\circ$ ; 3)  $30^\circ$ ; 4)  $45^\circ$ ; 5)  $60^\circ$ ; 6)  $75^\circ$ ; 7)  $90^\circ$ ; b: 1)  $\varphi = 135^\circ$ ; 2)  $150^\circ$ ; 3)  $165^\circ$ ; 4)  $180^\circ$ .

ratio  $Nu_u/Nu_{st} = f(\tau^*)$  for various values of the velocity are given in Fig. 2.

With the passage of a certain time, which varies from zero at angles close to  $90^\circ$  and  $270^\circ$  to 30-40 sec in the vicinity of the bow part of the cylinder generatrix, the heat-transfer coefficient in every case tends to a steady-state value, which depends on the flow velocity. These values of the heat-transfer coefficient are not, in the strict sense, the steady values of  $\alpha$ , because the temperature of the cylinder continued to vary with time, but they should tend to the steady values of  $\alpha$ . A comparison of the quasisteady values of the heat-transfer coefficient determined experimentally with the theoretical relation of G. N. Kruzhilin for  $\alpha$  in the vicinity of the bow line of the cylinder [8]

$$Nu = 1.04 \cdot Re^{0.5} Pr^{1/3} \quad (9)$$

evinces satisfactory agreement between them in the investigated interval of numbers Re. The discrepancy between the theoretical and experimental results does not exceed 6-9%. This fact supports the validity of the adopted measurement procedure.

Taking the experimentally obtained quasisteady values of  $\alpha$  as the steady values of the heat-transfer coefficient, we represent the measurement results in dimensionless-group form (Fig. 3a and 3b).

For angles  $\varphi = 0 - \pi/2$

$$\frac{Nu_u}{Nu_{st}} = 1.0 + \frac{600 - 382\varphi}{\tau^*}; \quad (10)$$

and for  $\varphi = \pi/2 - \pi$

$$\frac{Nu_u}{Nu_{st}} = 1.0 + \frac{-500 + 331\varphi}{\tau^*}, \quad (11)$$

where  $\tau^* = \tau v/R$  corresponds to twice the value of the number Ho in the notation of [3].

The given relations are valid in the investigated range of numbers  $Re = 10^4 - 10^5$ . The ratio  $Nu_u/Nu_{st}$  does not depend on the number Re, evincing the identical nature of the process under steady and unsteady conditions. This conclusion is consistent with the results obtained for unsteady heat transfer associated with airflow in a pipe [9].

Using expressions (9) and (10), in particular, for the unsteady heat-transfer coefficient, in the vicinity of the bow generatrix of the cylinder we have

$$Nu_u = 1.04 \cdot Re^{0.5} Pr^{1/3} (600 + \tau^*)/\tau^*. \quad (12)$$

The error of determination of  $Nu_u/Nu_{st}$ , including the errors of measurement of the quantities and the error of the approximation (10), (11), amounts to roughly 12%.

The satisfactory agreement between the measured values and the values of  $Nu$  calculated according to the theoretical expression (9) [8] and the significant disparity between the nature of the time variation of  $\alpha_u$  at different points around the perimeter of the cylinder indicate the absence of any appreciable methodological errors.

Thus, in the cooling of a metal cylinder in an airflow, during the initial ten seconds of the process the heat-transfer coefficient greatly exceeds its steady value in the bow and aft regions and is practically constant from the start of the process in the vicinity of extreme lateral points of the perimeter.

The large time interval in which the variation of  $\alpha$  occurs cannot be attributed solely to stabilization of the boundary layer at the instant of insertion of the body into the flow. The deviation of the heat-transfer coefficient from the steady value can also be due to the presence of unsteady heat transfer in the body and boundary layer during a large part of the cooling process.

The observed nature of the time variation of the unsteady heat-transfer coefficient is similar to the nature of the variation of  $\alpha_u$  in experiments involving the cooling of a Dur-alumin cylinder [2]. The large variation of the heat-transfer coefficient [2] in the initial period of the process can be related to large initial heating and the small thickness of the cylinder wall.

The derived relations (10) and (11) for the unsteady heat-transfer coefficient are suitable for the investigation of a cylinder in the above-indicated ranges of variation of  $Re$ , the dimensionless time, and the angle  $\varphi$ .

#### LITERATURE CITED

1. B. M. Smol'skii, L. A. Sergeeva, and V. L. Sergeev, Unsteady Heat Transfer [in Russian], Nauka i Tekhnika, Minsk (1974).
2. L. I. Kudryashev and A. A. Smirnov, "Influence of thermal unsteadiness on the heat-transfer coefficient in the case of external flow past bodies," *Inzh.-Fiz. Zh.*, 4, No. 10 (1961).
3. A. L. Parnas, "Experimental study of unsteady heat transfer of a cylinder in a transverse airflow," in: Unsteady Heat- and Mass-Transfer Problems [in Russian] (A. V. Lykov and B. M. Smol'skii, eds.), Izd. ITMO Akad. Nauk BSSR, Minsk (1964), pp. 30-35.
4. G. A. Surkov and G. A. Vanitskaya, "Unsteady heat transfer between a hollow cylinder and an impinging cooling fluid," in: Heat and Mass Transfer Associated with Intense Radiative and Convective Heating [in Russian] (R. I. Soloukhin, ed.), Izd. ITMO Akad. Nauk BSSR, Minsk (1977), pp. 141-145.
5. V. I. Zhuk and A. S. Golosov, "Engineering methods for determining thermal boundary conditions from the data of temperature measurements," *Inzh.-Fiz. Zh.*, 29, No. 1, 45-50 (1975).
6. B. T. Chao and D. R. Jeng, "Unsteady stagnation point heat transfer," *Trans. ASME, Ser. C: J. Heat Transfer*, 87, No. 2, 221 (1965).
7. T. B. Zdanavichus, "Influence of the degree of flow turbulence on the heat transfer of a cylinder in transverse flow for  $Re$  up to  $10^6$ ," Author's Abstract of Candidate's Dissertation, Kaunas. Politekh. Inst., Kaunas (1975).
8. M. I. Mikheev and I. M. Mikheeva, Fundamentals of Heat Transfer [in Russian], Énergiya, Moscow (1973).
9. B. M. Galitseiskii et al., "Experimental study of unsteady heat transfer in a tube with variation of the mass flow of gas and heat flow," *Vesti Akad. Navuk BSSR, Ser. Fiz. Mat. Navuk*, No. 2, 56-76 (1967).